$= \dfrac{-3+i\sqrt{3}}{1+i\sqrt{3}} = \dfrac{-\sqrt{3}(-i+\sqrt{3})}{i(\sqrt{3}-i)} = \sqrt{3}i$ d'où $\arg\left(\dfrac{\omega^2-1}{\omega}\right) \equiv \dfrac{\pi}{2}(2\pi)$

Conclusion : $(OM_1) \perp (MM_2)$

OMM_1M_2 est un losange.

Solution 23 :

1°/ $M_1M_2M_3$ est un triangle équilatéral si :

$\Re(M_1, \dfrac{\pi}{3})(M_2) = M_3 \Leftrightarrow M_1M_2 = M_1M_3$ et $(\overrightarrow{M_1M_2}, \overrightarrow{M_1M_3}) \equiv \dfrac{\pi}{3}$

(1) : $\Re(M_1, \dfrac{\pi}{3})(M_2) = M_3 \Leftrightarrow (z_3 - z_1) = e^{i\frac{\pi}{3}}(z_2 - z_1)$

or $e^{i\frac{\pi}{3}} = \cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3} = \dfrac{1}{2} + i\dfrac{\sqrt{3}}{2} = -\bar{j} = -j^2$

d'où $(z_3 - z_1) = -j^2(z_2 - z_1) \Leftrightarrow z_3 - z_1 + j^2z_2 - j^2z_1 = 0$

$\Leftrightarrow -(1+j^2)z_1 + j^2z_2 + z_3 = 0$ or $1 + j + j^2 = 0$ d'où $-(1+j^2) = j$

d'où $jz_1 + j^2z_2 + z_3 = 0 \Leftrightarrow j(z_1 + jz_2 + j^2z_3) = 0$ car $j^3 = 1$

d'où $z_1 + jz_2 + j^2z_3 = 0$

(2) : $\Re(M_1, \dfrac{\pi}{3})(M_3) = M_2 \Leftrightarrow z_2 - z_1 = e^{i\frac{\pi}{3}}(z_3 - z_1)$

$\Leftrightarrow z_2 - z_1 = -j^2(z_3 - z_1) \Leftrightarrow j^2z_1 + z_1 - z_2 - j^2z_3 = 0$

$\Leftrightarrow -jz_1 - z_2 - j^2z_3 = 0 \Leftrightarrow j(z_1 + j^2z_2 + jz_3) = 0 \Leftrightarrow z_1 + j^2z_2 + jz_3 = 0$

2°/ a) Soit $P(z) = z^3 - (1+a+ia)z^2 + a(1+i+ia)z - ia^2$

$P(1) = 1 - (1+a+ia) + a(1+i+ia) - ia^2$

$= 1 - 1 - a - ia + a + ia + ia^2 - ia^2 = 0$

b) $P(z) = (z-1)(z^2 + bz + ia^2) = z^3 + bz^2 + ia^2z - z^2 - bz - ia^2$

$= z^3 + (b-1)z^2 + z(ia^2 - b) - ia^2$ d'où $b - 1 = -1 - a - ia$

$b = -a - ia$